Holographic superconductors with higher curvature corrections

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ABSTRACT: We study (3+1)-dimensional holographic superconductors in Einstein-Gauss-Bonnet gravity both numerically and analytically. It is found that higher curvature corrections make condensation harder. We give an analytic proof of this result, and directly demonstrate an analytic approximation method that explains the qualitative features of superconductors as well as giving quantitatively good numerical results. We also calculate conductivity and ω_g/T_c, for ω_g and T_c the gap in the frequency dependent conductivity and the critical temperature respectively. It turns out that the 'universal' behaviour of conductivity, $\omega_g/T_c \simeq 8$, is not stable to the higher curvature corrections. In the appendix, for completeness, we show our analytic method can also explain (2+1)-dimensional superconductors.

Contents

1 Introduction

It is often felt that the most remarkable discovery in string theory has been the AdS/CFT correspondence, [1], which has been further extended to the gauge/gravity correspondence [2]. Interestingly, the gauge/gravity correspondence may play an important role in condensed matter physics [3, 4]. In particular, the application of the gauge/gravity correspondence to superconductors has been intensively studied [5–18] (see recent lecture notes [3, 4] for complete references). It would be very exciting indeed if we could explain high temperature superconductivity from black hole physics. In addition, from the gravity perspective the existence of scalar condensation in black hole systems itself deserves further study in relation to the "no-hair" theorems and a better understanding of the dressing of horizons by quantum fields [5, 19].

Remarkably, on the gauge theory side, there is a puzzle. As is well known, the Mermin-Wagner theorem forbids continuous symmetry breaking in (2+1)-dimensions because of large fluctuations in lower dimensions. Nevertheless, holographic superconductors are found in (2+1)-dimensions. It is possible that fluctuations in holographic superconductors are suppressed because classical gravity corresponds to the large N limit. If this is true, then higher curvature corrections should suppress condensation. Of course, to examine whether or not the Mermin-Wagner theorem holds, we need to study 4-dimensional higher curvature gravity. Unfortunately, higher curvature gravity in 4 dimensions is not particularly illuminating: higher derivative terms in general introduce ghost degrees of

freedom [20], the exceptions being either Gauss-Bonnet or Lovelock gravity [21], in which specific combinations of the curvature tensors are used, or $f(R)$ gravity, [22], in which powers of the Ricci scalar only are used. Unfortunately, the former case is non-dynamical in 4 dimensions, and the latter case is conformally equivalent to scalar-tensor gravity, [23], and black hole solutions are therefore identical to the Einstein case [24].

To explore this issue, we instead study 5-dimensional Einstein-Gauss-Bonnet gravity, which gives a known generalization to the Schwarzschild black hole solution [25]. We would also like to investigate if the universal relation between the gap ω_g in the frequency dependent conductivity and the critical temperature T_c: $\omega_g/T_c \simeq 8$, found in [7], is stable under stringy corrections. In the case of the quark-gluon plasma, there is a universal shear viscosity to entropy density ratio $\eta/s = 1/4\pi$ [27], and there are several analyses investigating the stability of this universal relation [28–35] to higher curvature corrections. To the best of our knowledge, no corresponding analysis exists in the case of superconductors. Hence, we look at gap frequency at a given temperature numerically to explore its stability under higher curvature corrections.

To investigate the effect of the higher curvature corrections on the superconductor, we operate in the 'probe' limit, i.e. where the gravitational back reaction of the scalar and vector fields on the background geometry is neglected. At least for temperatures near the phase transition this should be a good approximation, and has been found to be valuable in the Einstein limit [6]. Ideally, one would like to have a full analytic description of the phase transition and condensation phenomena, and in this paper we take a modest first step in this direction. We first prove the existence of a bound on black hole temperature above which no condensation can occur. Since there is always an analytic solution with vanishing scalar, [6], we cannot similarly prove the existence of a nontrivial scalar solution below T_c, however, a simple matching method provides an approximate analytic solution which explains the phase transition behaviour and gives a very good approximation to the phase diagram. Indeed, we can calculate the critical temperature analytically within a few percent in the best case. In a sense, this is the most important result in our paper. Numerical methods complete the proof of condensation, and are clearly necessary for fully describing the properties of the fields and the details of the physics.

The organization of the paper is as follows. In section II, we introduce the model and numerically demonstrate the effect of the Gauss-Bonnet term on the superconductor. We find that stringy corrections make condensation harder. In section III, we present an analytic explanation of the superconductor. We can understand the qualitative features of the superconductor with a simple calculation. The analysis also gives fairly good numerical results. In section IV, we study the conductivity and show the universality is unstable under the stringy corrections. We conclude in section V. In the appendix, we present an analytic explanation of (2+1)-dimensional superconductors for completeness.

2　Gauss-Bonnet superconductors

In this section, we study the effect of Gauss-Bonnet term on the (3+1)-dimensional superconductor using the probe limit. In the probe limit, gravity and matter decouple and

the system reduces to the Maxwell field and the charged scalar field in the neutral black hole background.

We begin with the Einstein-Gauss-Bonnet action:

$$S = \int d^5x \sqrt{-g} \left[R + \frac{12}{L^2} + \frac{\alpha}{2} \left(R^{\mu\nu\lambda\rho} R_{\mu\nu\lambda\rho} - 4R^{\mu\nu} R_{\mu\nu} + R^2 \right) \right] , \qquad (2.1)$$

where g is the determinant of a metric $g_{\mu\nu}$ and $R_{\mu\nu\lambda\rho}$, $R_{\mu\nu}$ and R are the Riemann curvature tensor, Ricci tensor, and the Ricci scalar, respectively. We take the Gauss-Bonnet coupling constant α to be positive. Here, the negative cosmological constant term $-6/L^2$ is also introduced. The background solution we consider is a neutral black hole [25]:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2}(dx^2 + dy^2 + dz^2) \qquad (2.2)$$

where

$$f(r) = \frac{r^2}{2\alpha} \left[1 - \sqrt{1 - \frac{4\alpha}{L^2} \left(1 - \frac{ML^2}{r^4} \right)} \right] \qquad (2.3)$$

Here, M is a constant of integration related to the "ADM" mass of the black hole [26]. The position of the horizon defined by $f(r_H) = 0$ is at $r_H = (ML^2)^{1/4}$. In order to avoid a naked singularity, we need to restrict the parameter range as $\alpha \leq L^2/4$. Note that in the Einstein limit ($\alpha \to 0$), the solution (2.3) goes to $f(r) = \frac{r^2}{L^2} - \frac{M}{r^2}$, and L can be regarded as the curvature radius of asymptotic AdS region ($r \to \infty$). For general α, however, the solution (2.3) behaves as

$$f(r) \sim \frac{r^2}{2\alpha} \left[1 - \sqrt{1 - \frac{4\alpha}{L^2}} \right] , \qquad (2.4)$$

in the asymptotic region. Hence, we define the effective asymptotic AdS scale by

$$L_{\text{eff}}^2 = \frac{2\alpha}{1 - \sqrt{1 - \frac{4\alpha}{L^2}}} \rightarrow \begin{cases} L^2 , & \text{for } \alpha \to 0 \\ \\ \frac{L^2}{2} , & \text{for } \alpha \to \frac{L^2}{4} \end{cases} . \qquad (2.5)$$

The Hawking temperature is given by

$$T = \frac{1}{4\pi} f'(r) \bigg|_{r=r_H} = \frac{r_H}{\pi L^2} = \frac{M^{1/4}}{\pi L^{3/2}} , \qquad (2.6)$$

where a prime denotes derivative with respect to r. This will be interpreted as the temperature of the CFT.

In this background, we now consider a Maxwell field and a charged complex scalar field, with the action

$$S = \int d^5x \sqrt{-g} \left[-\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - |\nabla\psi - iA\psi|^2 - m^2 |\psi|^2 \right] . \qquad (2.7)$$

Taking a static ansatz, $A_\mu = (\phi(r), 0, 0, 0, 0)$ and $\psi = \psi(r)$, the equation of motion for $\phi(r)$ becomes

$$\phi'' + \frac{3}{r}\phi' - \frac{2\psi^2}{f}\phi = 0 \qquad (2.8)$$

where without loss of generality ψ can be taken to be real, and satisfies

$$\psi'' + \left(\frac{f'}{f} + \frac{3}{r}\right)\psi' + \left(\frac{\phi^2}{f^2} - \frac{m^2}{f}\right)\psi = 0 \,. \qquad (2.9)$$

Note that the Maxwell equations imply that the phase of ψ must be constant, which is set to zero by a residual gauge for A_μ.

We now want to solve (2.8) and (2.9) for the scalar and vector field. For the main part of this paper, we choose to set the mass of the scalar field to be $m^2 = -3/L^2$, so that the mass remains the same as we vary α. Note however, that because of the variation of the effective asymptotic AdS curvature, (2.5), with α relative to L means that this mass actually *increases* (i.e. becomes less negative) with respect to the asymptotic AdS scale. On the other hand, while setting $m^2 = -3/L_{\text{eff}}^2$ has the advantage of fixing the mass relative to the asymptotic AdS scale, this mass now varies with respect to the physical measurables of black hole mass and temperature as we vary α. Since condensation is a temperature dependent phenomenon, we believe that fixing the scalar mass with respect to the black hole is the correct physical choice, however, we have also checked that for the alternative choice of mass the same qualitative features occur as we vary α.

In order to solve our equations we need to impose regularity at the horizon and the AdS boundary:

- Regularity at the horizon gives two conditions:

$$\phi(r_H) = 0, \qquad \psi(r_H) = -\frac{4}{3}r_H\psi'(r_H) \,. \qquad (2.10)$$

- Asymptotically ($r \to \infty$) the solutions are found to be:

$$\phi(r) = \mu - \frac{\rho}{r^2}, \qquad \psi = \frac{C_-}{r^{\lambda_-}} + \frac{C_+}{r^{\lambda_+}}, \qquad (2.11)$$

where $\lambda_\pm = 2 \pm \sqrt{4 - 3\left(\frac{L_{\text{eff}}}{L}\right)^2}$. Here, μ and ρ are interpreted as a chemical potential and charge density, respectively. Note that these are not entirely free parameters, as there is a scaling degree of freedom in the equations of motion. As in [6], we impose that ρ is fixed, which determines the scale of this system. For ψ, both of these falloffs are normalizable, so we can impose the condition either C_- or C_+ vanish. We take $C_- = 0$, for simplicity.

According to the AdS/CFT correspondence, we can interpret $\langle \mathcal{O} \rangle \equiv C_+$, where $\mathcal{O}$ is the operator dual to the scalar field. Thus, we are going to calculate the condensate $\langle \mathcal{O} \rangle$ for fixed charge density. The results are shown in figure 1. From figure 1, we see the GB term makes the condensation gap larger. We also see that the Chern-Simons limit shows

a slightly different dependence of the condensate on temperature. This can be understood from the behaviour of gravity near the horizon. In the Chern-Simons limit, $\alpha = L^2/4$, we get

$$f(r) = \frac{2r^2}{L^2} \left(1 - \frac{\sqrt{ML^2}}{r^2} \right) . \tag{2.12}$$

Hence, the correction to the AdS quadratic gravitational potential dependence is simply a constant instead of a $1/r^2$ dependence, leading to more gentle tidal behaviour. The process of scalar condensation (or the formation of scalar hair) can be understood as arising in part from the 'negative' mass of the scalar field, but also as arising from the potential well that occurs near the horizon. For black holes with large mass, this well is too broad and shallow to allow for the formation of a nonzero scalar, however, for small black hole mass, the strong curvature near the horizon is amenable to condensation. (Refer to the analytic arguments in section III which show how the behaviour of the gravitational potential interacts with features of the scalar condensate.) At some stage further decreasing the mass of the black hole does not alter the shape of the condensate much, as the scalar is already sampling regions of strong curvature. However, the CS limit has a rather different and smoother profile near the horizon, therefore it is not surprising that decreasing the black hole mass in this case has more impact on the details of the scalar field.

Numerically, we found that increasing α resulted in a decrease of the critical temperature: $T_c = 0.198 \rho^{1/3}$ for $\alpha = 0.0001$, $T_c = 0.186 \rho^{1/3}$ for $\alpha = 0.1$, $T_c = 0.171 \rho^{1/3}$ for $\alpha = 0.2$, and $T_c = 0.158 \rho^{1/3}$ for $\alpha = 0.25$ (see also figure 2). Thus the effect of α is to make it harder for scalar hair to form. Changing the scalar mass to $m^2 = -3/L_{\text{eff}}^2$ gives a similar, though less marked, behaviour, for example $T_c = 0.181 \rho^{1/3}$ for $\alpha = 0.2$. We can therefore conclude, as expected, that the higher curvature corrections make it harder for the scalar hair to form. One can expect this tendency to be the same even in (2+1)-dimensions, however, it remains obscure to what extent this suppression affects the physics of holographic superconductors in (2+1)-dimensions.

We have thus numerically verified that Gauss-Bonnet superconductors exist. However, we would ideally like to have an analytic understanding of condensation to back up this numerical work. This is what we now turn to.

3 Superconductors in a nutshell

Although in the previous section we used numerical integration to explicitly demonstrate the condensation phenomenon, ideally we would like to obtain an analytic understanding in parallel. Since our equations are nonlinear and coupled, we cannot derive analytic solutions in closed form, however we can deduce a great deal of information analytically. We first prove the nonexistence of condensation for large T before explicitly deriving the phase diagram analytically by using approximate solutions.

Note that the trivial solution to (2.8) and (2.9)

$$\phi = \phi_0(r) = \frac{\rho}{r_H^2} \left(1 - \frac{r_H^2}{r^2} \right) \tag{3.1}$$

$$\psi \equiv 0 \tag{3.2}$$
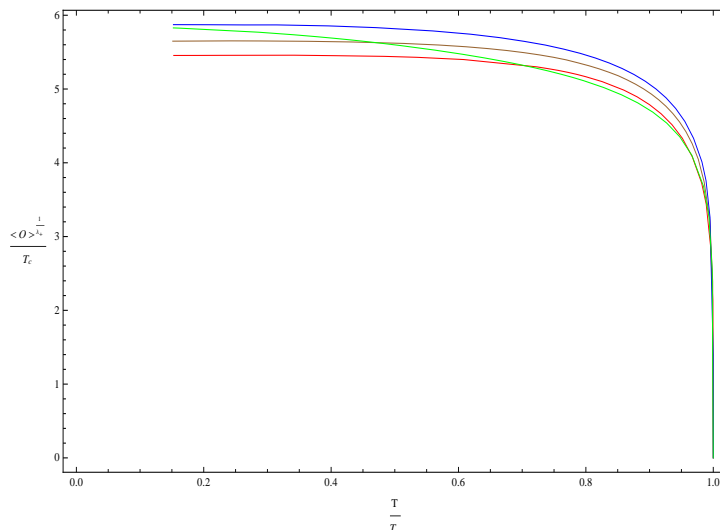
Figure 1. The condensate as a function of temperature for various values of α. The (lowest) red line is for $\alpha = 0.0001$, the middle brown plot is $\alpha = 0.1$, the top blue line is $\alpha = 0.2$ and the remaining line intersecting the other three in green is the Chern-Simons limit $\alpha = 0.25$. Note that while the generic Einstein-Gauss-Bonnet behaviour is to level out for $T \leq T_c/2$, the Chern-Simons limit has a much stronger variation of the condensate with temperature.

always exists. We will now prove that there is a temperature above which this is the only solution.

First consider the ϕ equation (2.8). Let $\phi(r) = \phi_0(r) + \delta\phi$, where $\phi_0(r)$ is defined above. Then (2.8) implies

$$\left(r^3 \delta\phi'\right)' \geq 0 \tag{3.3}$$

however, as $r \to \infty$, $r^3\phi' \to 2\rho = r^3\phi_0'$, hence $r^3\delta\phi' \to 0$ at infinity, and using $\delta\phi = 0$ at r_H we have that $\delta\phi' \leq 0$. Hence

$$\phi(r) \leq \phi_0(r) . \tag{3.4}$$

Next consider the scalar field, and define the variable $X = r\psi$:

$$X'' + \left(\frac{f'}{f} + \frac{1}{r}\right) X' + \left(\frac{\phi^2}{f^2} + \frac{3}{L^2 f} - \frac{f'}{rf} - \frac{1}{r^2}\right) X = 0 \tag{3.5}$$

Now, the boundary conditions at the horizon imply $X_H' = X_H/4r_H$, and at infinity, $rfX' \to 0$, thus the existence of a condensate requires a turning point in X, $X'(r_T) = 0$, with $X'' < 0$ for $X > 0$. This in turn requires

$$\frac{\phi_0^2(r_T)}{f(r_T)} + \frac{3}{L^2} - \frac{f'(r_T)}{r_T} - \frac{f(r_T)}{r_T^2} > \frac{\phi^2(r_T)}{f(r_T)} + \frac{3}{L^2} - \frac{f'(r_T)}{r_T} - \frac{f(r_T)}{r_T^2} > 0 \tag{3.6}$$

at the turning point. By inputting the form of $\phi_0(r)$, it is easy to see that if M is too large, this inequality can never be satisfied, as the combination of ϕ and the geometry to the l.h.s. of (3.6) is always negative. This gives a loose upper bound on the critical temperature as shown in figure 2. (For $\alpha = 0$ we need to use the fact that $\int_{r_T}^{\infty} (rfX')' = 0$ to bound M.
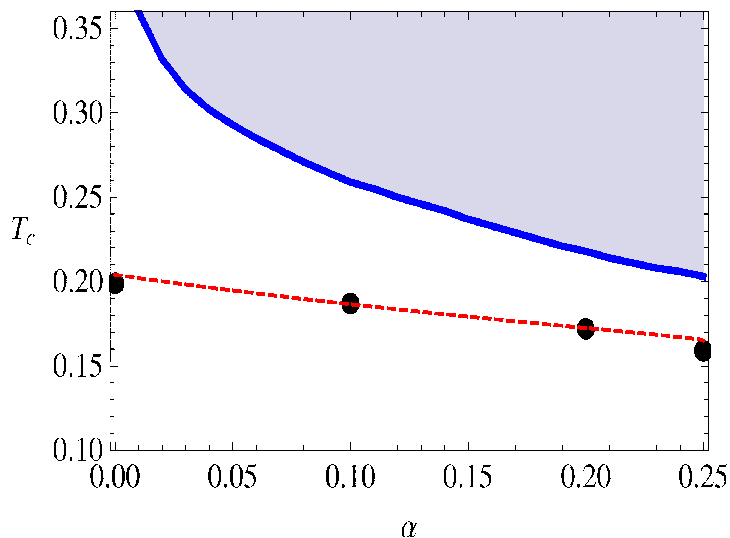
Figure 2. A comparison of analytic and numerical results. The shaded region is that in which the geometry forbids the possibility of a scalar condensate from (3.6). The dashed line indicates the analytic approximation of the value of T_c obtained by matching methods, (3.27). The data points are the exact numerical results. For simplicity ρ and L have been set to 1.

This also gives a tighter analytical bound for nonzero α, however, the above argument is more direct.)

We have thus numerically verified that Gauss-Bonnet superconductors exist. Having shown that there is a critical temperature below which there is no barrier to condensation, we will now show we can understand the essential features of condensation by using approximation techniques.

Once again, let us change variables and set $z = \frac{r_H}{r}$. Under this transformation equations (2.8) and (2.9) become

$$\phi'' - \frac{1}{z}\phi' - \frac{r_H^2}{z^4}\frac{2\psi^2}{f}\phi = 0 \tag{3.7}$$

$$\psi'' + \left(\frac{f'}{f} - \frac{1}{z}\right)\psi' + \frac{r_H^2}{z^4}\left(\frac{\phi^2}{f^2} + \frac{3}{L^2 f}\right)\psi = 0 \tag{3.8}$$

where a prime now denotes $\frac{d}{dz}$. The region $r_H < r < \infty$ now corresponds to $0 < z < 1$. The boundary conditions now become:

- Regularity at the horizon $z = 1$ gives

$$\phi(1) = 0, \qquad \psi'(1) = \frac{3}{4}\psi(1). \tag{3.9}$$

- In the asymptotic AdS region: $z \to 0$, the solutions are

$$\phi = \mu - qz^2, \qquad \psi = D_- z^{\lambda_-} + D_+ z^{\lambda_+}, \tag{3.10}$$

where $\lambda_\pm$ is the same as in equation (2.11). As boundary conditions, we fix qr_H^2 and take D_- to be zero.

We now find leading order solutions near the horizon and asymptotically, say $1 \geq z > z_m$ and $z_m > z \geq 0$, and then match these smoothly at the intermediate point, z_m. As a consequence, we will demonstrate the phase transition phenomenon directly, and derive an (approximate) analytic expression for the critical temperature. Moreover, we will have a much better analytical understanding of α dependence of the critical temperature, as the proof above only gives a loose bound on the critical temperature and only indirect access to an expression.

3.1 Solution near the horizon: $z = 1$

We can expand ϕ and ψ in a Taylor series near the horizon as:

$$\phi(z) = \phi(1) - \phi'(1)(1-z) + \frac{1}{2}\phi''(1)(1-z)^2 + \cdots \tag{3.11}$$

$$\psi(z) = \psi(1) - \psi'(1)(1-z) + \frac{1}{2}\psi''(1)(1-z)^2 + \cdots \tag{3.12}$$

From (3.9), we have $\phi(1) = 0$ and $\psi'(1) = \frac{3}{4}\psi(1)$, and without loss of generality we take $\phi'(1) < 0$, $\psi(1) > 0$ to have $\phi(z)$ and $\psi(z)$ positive. Expanding (3.7) near $z = 1$ gives:

$$\begin{aligned}
\phi''(1) &= \frac{1}{z}\phi' \Big|_{z=1} + \frac{r_H^2}{z^4}\frac{2\psi^2}{f}\phi \Big|_{z=1} \\
&= \phi'(1) - \frac{2r_H^2\psi(1)^2}{z^4(1-z)f'(1)}\left(-\phi'(1)(1-z) + \frac{1}{2}\phi''(1)(1-z)^2 + \cdots\right)\Big|_{z=1} \\
&= \left(1 - \frac{L^2}{2}\psi(1)^2\right)\phi'(1)
\end{aligned} \tag{3.13}$$

Thus, we get the approximate solution

$$\phi(z) = -\phi'(1)(1-z) + \frac{1}{2}\left(1 - \frac{L^2}{2}\psi(1)^2\right)\phi'(1)(1-z)^2 + \cdots \tag{3.14}$$

Similarly, from (3.8), the 2nd order coefficients of ψ can be calculated as

$$\begin{aligned}
\psi''(1) &= \frac{1}{z}\psi'\Big|_{z=1} - \frac{z^4 f'\psi' + 3\frac{r_H^2}{L^2}\psi}{z^4 f}\Big|_{z=1} - \frac{r_H^2\phi^2}{z^4 f^2}\psi\Big|_{z=1} \\
&= \psi'(1) - \frac{4z^3 f'\psi' + z^4 f''\psi' + z^4 f'\psi'' + 3\frac{r_H^2}{L^2}\psi'}{4z^3 f + z^4 f'}\Big|_{z=1} - \frac{r_H^2\psi\left(-\phi'(1)(1-z)+\cdots\right)^2}{f'(1)^2(1-z)^2}\Big|_{z=1} \\
&= -\frac{5}{4}\psi'(1) + 8\frac{\alpha}{L^2}\psi'(1) - \psi''(1) - \frac{L^4}{16r_H^2}\phi'(1)^2\psi(1)
\end{aligned} \tag{3.15}$$

where we used l'Hôpital's rule at the second term in the second line. Thus, we get

$$\psi''(1) = \left(-\frac{5}{8} + \frac{4\alpha}{L^2}\right)\psi'(1) - \frac{L^4}{32r_H^2}\phi'(1)^2\psi(1) \tag{3.16}$$

After eliminating $\psi'(1)$ from above equation by using eq. (3.9), we find an approximate solution near the horizon as

$$\psi(z) = \frac{1}{4}\psi(1) + \frac{3}{4}\psi(1)z + \left(-\frac{15}{64} + \frac{3\alpha}{2L^2} - \frac{L^4}{64r_H^2}\phi'(1)^2\right)\psi(1)(1-z)^2 + \cdots \tag{3.17}$$

3.2 Solution near the asymptotic AdS region: $z = 0$

From (3.10), ϕ and ψ in the asymptotic region are given by

$$\phi(z) = \mu - qz^2 \,, \qquad \psi(z) = D_+ z^{\lambda_+} \tag{3.18}$$

where qr_H^2 is fixed and we have set $D_- = 0$ from the boundary condition.

3.3 Matching and phase transition

Now we will match the solutions (3.14), (3.17) and (3.18) at z_m. Interestingly, allowing z_m to be arbitrary does not change qualitative features of the analytic approximation, more importantly, it does not give a big difference in numerical values, therefore for simplicity in demonstrating our argument we will take $z_m = 1/2$. In order to connect our two asymptotic solutions smoothly, we require the following 4 conditions:

$$\mu - \frac{1}{4}q = \frac{1}{2}b - \frac{1}{8}b\left(1 - \frac{L^2}{2}a^2\right) \tag{3.19}$$

$$-q = -b + \frac{1}{2}b\left(1 - \frac{L^2}{2}a^2\right) \tag{3.20}$$

$$D_+\left(\frac{1}{2}\right)^{\lambda_+} = \frac{5}{8}a + \frac{1}{4}a\left(-\frac{15}{64} + \frac{3\alpha}{2L^2} - \frac{L^4}{64r_H^2}b^2\right) \tag{3.21}$$

$$2\lambda_+ D_+\left(\frac{1}{2}\right)^{\lambda_+} = \frac{3}{4}a - a\left(-\frac{15}{64} + \frac{3\alpha}{2L^2} - \frac{L^4}{64r_H^2}b^2\right) \tag{3.22}$$

where we have set $\psi(1) \equiv a$ and $-\phi'(1) \equiv b$ ($a, b > 0$) for clarity. Now, the AdS/CFT dictionary gives a relation $\langle \mathcal{O} \rangle \equiv L D_+ r_H^{\lambda_+} L^{-2\lambda_+}$, hence we need to compute D_+. From (3.21) and (3.22) we obtain

$$D_+ = \frac{13}{8}\frac{2^{\lambda_+}}{\lambda_+ + 2}\, a \,. \tag{3.23}$$

Using (3.19) and (3.20), a is expressed by

$$a^2 = \frac{4q}{L^2 b}\left(1 - \frac{b}{2q}\right) \,, \tag{3.24}$$

where b is obtained from (3.21) and (3.22) assuming $a \neq 0$ (i.e. the scalar solution is non-trivial) as:

$$b = 8\frac{r_H}{L^2}\sqrt{\frac{5\lambda_+ - 3}{2(\lambda_+ + 2)} - \frac{15}{64} + \frac{3\alpha}{2L^2}} \,. \tag{3.25}$$

Now we go back to the original variable, r, and compare the results with those in [7]. First of all, we should note the relation $\rho = q\, r_H^2$. We also define $\tilde{b}$ by $b = \tilde{b}r_H/L^2$. Using the Hawking temperature $T = \frac{r_H}{\pi L^2}$, we can rewrite (3.24) as

$$a^2 = \frac{2}{L^2}\frac{T_c^3}{T^3}\left(1 - \frac{T^3}{T_c^3}\right) \,, \tag{3.26}$$

where we have defined T_c as

$$T_c = \left(\frac{2\rho}{\tilde{b}L}\right)^{1/3}\frac{1}{\pi L}\ . \tag{3.27}$$

We can now read off the expectation value $\langle\mathcal{O}\rangle$ from (3.23) and (3.26) as:

$$\frac{\langle\mathcal{O}\rangle^{\frac{1}{\lambda_+}}}{T_c} = 2\pi\left(\frac{13}{8}\frac{\sqrt{2}}{\lambda_+ + 2}\right)^{\frac{1}{\lambda_+}}\frac{T}{T_c}\left[\ \frac{T_c^3}{T^3}\left(1 - \frac{T^3}{T_c^3}\right)\ \right]^{\frac{1}{2\lambda_+}}\ , \tag{3.28}$$

where we have normalized by the critical temperature to obtain a dimensionless quantity. We find that $\langle\mathcal{O}\rangle$ is zero at $T = T_c$, the critical point, and condensation occurs for $T < T_c$. We also see a behaviour $\langle\mathcal{O}\rangle \propto (1 - T/T_c)^{1/2}$ which is a typical mean field theory result for a the second order phase transition.

Next, we evaluate the critical temperature from (3.27). The value of T_c is $0.201\rho^{1/3}/L$ when the Gauss-Bonnet term is absent, this should be compared with the numerical result $T_c = 0.198\rho^{1/3}/L$ in [7]. We therefore see that our analytic approximation is good. Moreover, as α increases to $0.1, 0.2$ and 0.25, T_c decreases to $0.196, 0.191$ and 0.188 respectively, which is in good agreement with our numerical results.

Thus, we have (approximately) reproduced our numerical results from a simple analytic calculation. In particular, we have calculated extremely good estimates of the critical temperatures, and revealed how the structure of the interaction term has produced the phase transition.

4 Conductivity and universality

We now calculate the conductivity, σ, of our boundary theory. In [7], the conductivity for various cases was calculated and it was found that there is a universal relation

$$\frac{\omega_g}{T_c} \simeq 8\,, \tag{4.1}$$

with deviations of less than 8 %. The purpose of this section is to examine if this universality holds in the presence of stringy corrections.

As A_μ in the bulk corresponds to the four-current J_μ on the CFT boundary, we can calculate the conductivity by considering perturbation of A_μ. The spatial components of A_μ are decomposed into longitudinal and transverse modes: $A_i = (\partial_i\chi, A_i^\perp)$. These linearized perturbations are decoupled from each other and can be studied separately. The linearized equation of motion for $A_i^\perp(t, r, x^i) = A(r)e^{i\mathbf{k}\cdot\mathbf{x} - i\omega t}e_i$, which corresponds to the current density, is

$$A'' + \left(\frac{f'}{f} + \frac{1}{r}\right)A' + \left(\frac{\omega^2}{f^2} - \frac{\mathbf{k^2}}{r^2 f} - \frac{2}{f}\psi^2\right)A = 0\ . \tag{4.2}$$

We solve this under the following boundary conditions near the horizon:

$$A(r) \sim f(r)^{-i\frac{\omega}{4r_H}}\ , \tag{4.3}$$

which corresponds to no outgoing radiation at the horizon. In the asymptotic AdS region $(r \to \infty)$, the general solution takes the form

$$A = A^{(0)} + \frac{A^{(2)}}{r^2} + \frac{A^{(0)}(\omega^2 - \mathbf{k}^2)L_{\text{eff}}^2}{2} \frac{\log \Lambda r}{r^2} \tag{4.4}$$

where $A^{(0)}$, $A^{(2)}$ and Λ are arbitrary integration constants. Note the appearance of the arbitrary scale Λ, which leads to a logarithmic divergence in the Green's function, as explained in [7]. Since this can be removed by an appropriate boundary counterterm, this scale will disappear from the results.

From linear response theory, the conductivity can be calculated by the formula

$$\sigma(\omega) = \frac{1}{i\omega} G^R(\omega, k = 0) , \tag{4.5}$$

where k is the wavenumber. The retarded Green function G^R can be calculated through the AdS/CFT correspondence [36] as:

$$G^R = -\lim_{r \to \infty} f(r) r A A' . \tag{4.6}$$

Thus, by using the solution (4.4), the conductivity is given by

$$\sigma = \left. \frac{2A^{(2)}}{i\omega A^{(0)}} \right|_{\mathbf{k}=\mathbf{0}} + \frac{i\omega}{2} . \tag{4.7}$$

We therefore need to solve (4.2) numerically with the boundary condition (4.3) to obtain $A^{(0)}$ and $A^{(2)}$ asymptotically.

The plots in figures 3−6 show the results of this numerical integration for $\alpha = 0.0001, 0.1, 0.2$ and 0.25 at temperatures $T/T_c \approx 0.152, 0.151, 0.152$ and 0.152, respectively. The red line represents the real part, and blue line the imaginary part of σ. Taking look at the imaginary part of the conductivity, we see a pole exists at $\omega = 0$. From the Kramers-Kronig relations, this implies the real part of the conductivity contains a delta function.

Clearly, the real part of the conductivity shows a frequency gap which indicates a gap in the spectrum of charged excitations. As α increases, the gap frequency (normalized by T_c) becomes large. As we noticed that condensation is an increasing function of α this tendency is consistent with the conventional relation $\omega_g \propto \langle \mathcal{O} \rangle$. We see the universal relation $\frac{\omega_g}{T_c} \approx 8$ found in [7] is unstable in the presence of the Gauss-Bonnet correction. We have also checked that this conclusion is not affected by choosing the alternative scalar mass, $M^2 = -3/L_{\text{eff}}^2$.

5   Conclusion

We have studied holographic superconductors in the presence of Gauss-Bonnet corrections to the gravitational action. Motivated by the Mermin-Wagner theorem, we have investigated if the higher derivative corrections suppress the phase transition or not. We numerically solved the system in the probe limit and obtained phase diagrams for various
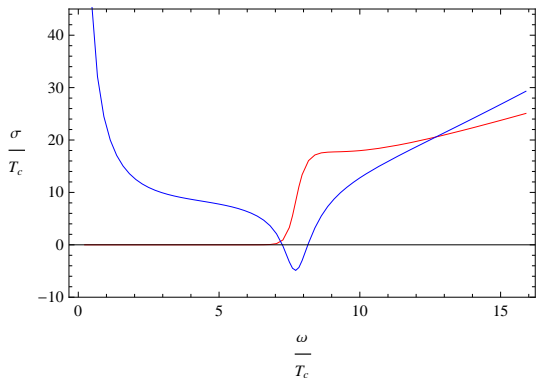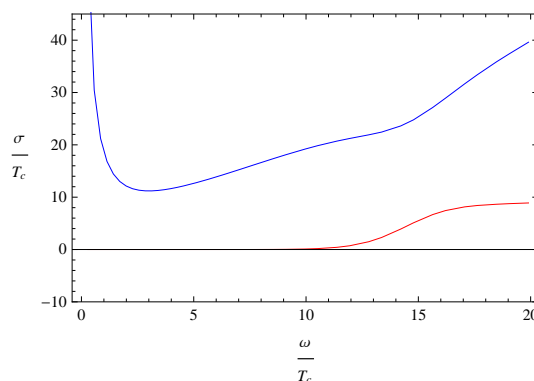
Figure 3. Conductivity for $\alpha = 0.0001$ case.
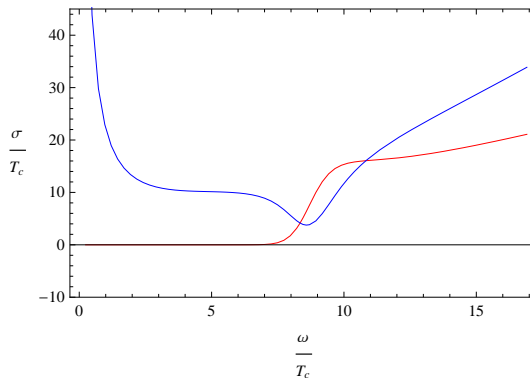
Figure 4. Conductivity for $\alpha = 0.1$ case.
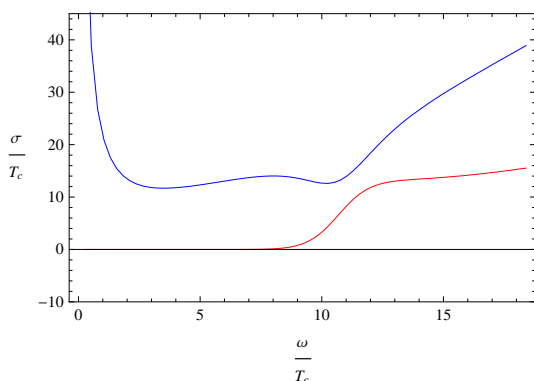
Figure 5. Conductivity for $\alpha = 0.2$ case.

Figure 6. Conductivity for $\alpha = 0.25$ case.

Gauss-Bonnet couplings α, and calculated the critical temperatures. As we increase α, the critical temperature decreases, thus it turns out that stringy corrections make condensation harder. However, we did not reach the point that the critical temperature of the transition vanishes for changing α. We would also expect this to apply to the (2+1)-dimensional case, however, it is beyond the scope of this paper to determine if this could destroy holographic superconductors in (2+1)-dimensions.

To understand phase transition phenomena, we also conducted an analytic analysis of the coupled nonlinear equations, finding an approximate analytic solution. In spite of the apparent crudity of this approximation, we have analytically demonstrated the phase transition. Surprisingly, it turned out that the analytical method gave good agreement with the numerical results. In particular, we have calculated the critical temperature analytically. We obtained $T_c = 0.201\rho^{1/3}$, which is close to the numerical result $T_c = 0.198\rho^{1/3}$ for $\alpha \to 0$. We also applied the same method to the (2+1)-dimensional superconductor, presented in the appendix. The resultant critical temperature was $T_c = 0.103\sqrt{\rho}$, which should be compared with the numerical result $T_c = 0.118\sqrt{\rho}$ [6].

Our other purpose was to examine the universality of the gap frequency to the crit-

ical temperature ratio. By calculating conductivity, we found the universal behaviour of conductivity $\omega_g/T_c \simeq 8$ was unstable to the stringy corrections.

There are many issues to be investigated further. The obvious next step is to incorporate back reaction, which is particularly important in the low temperature regime which corresponds to small black holes. In that case, the stability of black holes should be considered [37–40]. Although we have investigated the stability of the superconductor under stringy corrections, it is also intriguing to study the dynamical stability of the condensation phase, as well as other aspects of superconductors [41–44].

Acknowledgments

A Analytical approach to (2+1)-dimensional superconductors

In the main text, we have shown a simple analytic treatment gives a good explanation of superconductivity. Here for completeness, we show (2+1)-dimensional superconductors can be also explained using the same method.

In the 4-dimensional case, we have the following equations

$$\phi'' + \frac{2}{r}\phi' - \frac{2\psi^2}{f}\phi = 0\,, \tag{A.1}$$

$$\psi'' + \left(\frac{f'}{f} + \frac{2}{r}\right)\psi' + \left(\frac{\phi^2}{f^2} - \frac{m^2}{f}\right)\psi = 0\,, \tag{A.2}$$

where now

$$f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_H^3}{r^3}\right) \tag{A.3}$$

with $r_H = (ML^2)^{1/3}$. We set the mass of the scalar field, $m^2 = -2/L^2$, as in [6]. By changing to the z variable as before, $z = \frac{r_H}{r}$, (A.1) and (A.2) become

$$\phi'' - \frac{2L^2\psi^2}{z^2(1-z^3)}\phi = 0 \tag{A.4}$$

$$\psi'' - \frac{2+z^3}{z(1-z^3)}\psi' + \left(\frac{L^4\phi^2}{r_H^2(1-z^3)^2} + \frac{2}{z^2(1-z^3)}\right)\psi = 0 \tag{A.5}$$

where a prime now denotes $\frac{d}{dz}$. Next we consider the boundary conditions with these new variables. Regularity at the horizon, $z = 1$, requires

$$\phi(1) = 0\,, \qquad \psi'(1) = \frac{2}{3}\psi(1)\,, \tag{A.6}$$

and the asymptotic solution in the AdS region, $z \to 0$, reads

$$\phi = \mu - qz, \qquad \psi = C_1 z + C_2 z^2 . \tag{A.7}$$

As in [6] we fix the charge qr_H and take C_1 to be zero.

We now find an approximate solution around both $z = 1$ and $z = 0$ using Taylor expansion as before, then connect these solutions between $z = 1$ and $z = 0$.

A.1 Solution near the horizon: $z = 1$

We expand ϕ and ψ as

$$\phi(z) = \phi(1) - \phi'(1)(1 - z) + \frac{1}{2}\phi''(1)(1 - z)^2 + \cdots \tag{A.8}$$

$$\psi(z) = \psi(1) - \psi'(1)(1 - z) + \frac{1}{2}\psi''(1)(1 - z)^2 + \cdots \tag{A.9}$$

From the boundary condition (A.6), $\phi(1) = 0$ and $\psi'(1) = \frac{2}{3}\psi(1)$, and we again set $\phi'(1) < 0$ and $\psi(1) > 0$ for positivity of $\phi(z)$ and $\psi(z)$.

First we compute the 2$^{\text{nd}}$ order coefficient ϕ using (A.4) as

$$\phi'' \Big|_{z=1} = \frac{2L^2\psi^2}{z^2(1 - z^3)}\phi \Big|_{z=1} = -\frac{2}{3}L^2\phi'(1)\psi(1)^2 > 0 , \tag{A.10}$$

giving

$$\phi(z) = -\phi'(1)(1 - z) - \frac{1}{3}L^2\psi(1)^2\phi'(1)(1 - z)^2 + \cdots . \tag{A.11}$$

The 2nd derivative of ψ is calculated similarly as

$$
\begin{aligned}
\psi'' \Big|_{z=1} &= \frac{2 + z^3}{z(1 - z^3)}\psi' \Big|_{z=1} - \frac{2}{z^2(1 - z^3)}\psi \Big|_{z=1} - \frac{L^4\phi^2}{r_H^2(1 - z^3)^2}\psi \Big|_{z=1} \\
&= \frac{(z^4 + 2z)\psi'' + 4z^3\psi'}{2z - 5z^4} \Big|_{z=1} - \frac{L^4}{9r_H^2}\phi'(1)^2\psi(1) \\
&= -\psi''(1) - \frac{4}{3}\psi'(1) - \frac{L^4}{9r_H^2}\phi'(1)^2\psi(1) \tag{A.12}
\end{aligned}
$$

Thus

$$\psi''(1) = -\frac{2}{3}\psi'(1) - \frac{L^4}{18r_H^2}\phi'(1)^2\psi(1) . \tag{A.13}$$

Using (A.6) to eliminate ψ', we find

$$\psi(z) = \frac{1}{3}\psi(1) + \frac{2}{3}\psi(1)z - \frac{2}{9}\left(1 + \frac{L^4}{8r_H^2}\phi'(1)^2\right)\psi(1)(1 - z)^2 + \cdots \tag{A.14}$$

A.2 Solution near the asymptotic AdS region: $z = 0$

We expand ϕ and ψ, making use of asymptotic solutions (A.7), as

$$\phi(z) = \mu - qz + \frac{1}{2}\phi''(0)z^2 + \cdots \tag{A.15}$$

$$\psi(z) = C_2 z^2 + \cdots \tag{A.16}$$

where we have used $C_1 = 0$.

Then the 2nd derivative of ϕ is given by

$$\phi'' \big|_{z=0} = \frac{2L^2\psi^2}{z^2(1-z^3)}\phi \bigg|_{z=0} = 0 \tag{A.17}$$

and we get simply

$$\phi(z) = \mu - qz, \qquad \psi(z) = C_2 z^2 \tag{A.18}$$

where qr_H is fixed.

A.3 Matching and phase transition

As before, we connect the solutions (A.11), (A.14) and (A.18) at $z = \frac{1}{2}$. In order to connect those solutions smoothly, we require the following 4 conditions:

$$\mu - \frac{1}{2}q = \frac{1}{2}b + \frac{L^2}{12}a^2 b \tag{A.19}$$

$$-q = -b - \frac{L^2}{3}a^2 b \tag{A.20}$$

$$\frac{1}{4}C_2 = \frac{11}{18}a - \frac{L^4}{144r_H^2}ab^2 \tag{A.21}$$

$$C_2 = \frac{8}{9}a + \frac{L^4}{36r_H^2}ab^2 \tag{A.22}$$

where $\psi(1) \equiv a$ and $-\phi'(1) \equiv b$, with $(a, b > 0)$ as before. Eliminating $a^2 b$ from (A.19) and (A.20) gives

$$\mu = \frac{3}{4}q + \frac{1}{4}b . \tag{A.23}$$

From (A.19) and (A.20), we can also deduce

$$a^2 = \frac{12}{L^2 b}(q - \mu) . \tag{A.24}$$

The above relation alludes to phase transitions, namely, given q, μ has a maximum value when we assume the non-trivial solution $a \neq 0$. Substituting the relation (A.23) into (A.24), we have

$$a = \frac{\sqrt{3}}{L}\sqrt{\frac{q}{b}}\sqrt{1 - \frac{b}{q}} . \tag{A.25}$$

To relate this result to the expectation value of the dimension 2 operator $\langle \mathcal{O}_2 \rangle = \sqrt{2}C_2 r_H^2/L^3$, we eliminate ab^2 from (A.21) and (A.22) to obtain

$$C_2 = \frac{5}{3}a \ . \tag{A.26}$$

Similarly, eliminating C_2 from (A.21) and (A.22) gives

$$a\left(b^2 - 28\frac{r_H^2}{L^4}\right) = 0\,, \tag{A.27}$$

which determines $b = 2\sqrt{7}r_H/L^2$ provided $a \neq 0$.

Now we are in a position to reveal the phase transition phenomenon in this simple system. Noting the relation $\rho = q\, r_H$, and using the Hawking temperature: $T = \frac{3r_H}{4\pi L^2}$, $\langle \mathcal{O}_2 \rangle$ can be expressed by

$$\langle \mathcal{O}_2 \rangle = \frac{80\pi^2}{9}\sqrt{\frac{2}{3}}T_c T\sqrt{1 + \frac{T}{T_c}}\sqrt{1 - \frac{T}{T_c}} \tag{A.28}$$

where T_c is defined as

$$T_c = \frac{3\sqrt{\rho}}{4\pi L\sqrt{2\sqrt{7}}} \tag{A.29}$$

We see that $\langle \mathcal{O}_2 \rangle$ is zero at $T = T_c$, which is a critical point, and condensation occurs at $T < T_c$. The mean field theory result $\langle \mathcal{O}_2 \rangle \propto (1 - T/T_c)^{1/2}$ is also recovered. The value, (A.29), of T_c is evaluated as $0.103\sqrt{\rho}/L$. Comparing with the numerical result $0.118\sqrt{\rho}/L$ in [6], we find our analytic approximation is quantitatively good. Also the coefficient of $(1 - T/T_c)^{1/2}$ as $T \to T_c$ is now $101T_c^2$, while the numerical result is $144T_c^2$ [6], which means this approximation seems good.

One may wonder what happens if we change z_m. If we connect the solutions at z_m ($0 < z_m < 1$), the result is

$$\langle \mathcal{O}_2 \rangle = \frac{16\pi^2}{9}\frac{2 + z_m}{3z_m}\sqrt{\frac{3}{1 - z_m}}T_c T\sqrt{1 + \frac{T}{T_c}}\sqrt{1 - \frac{T}{T_c}} \tag{A.30}$$

where

$$T_c = \frac{3}{4\pi L}\sqrt{\frac{\rho}{\tilde{b}}}\,, \qquad \tilde{b} = \sqrt{\frac{4(1 + 5z_m)}{1 - z_m}} \tag{A.31}$$

In order to get the same value $T_c = 0.118\sqrt{\rho}/L$ as [6], we need to choose $z_m = 0.34$. For this value, the coefficient of $(1 - T/T_c)^{1/2}$ as $T \to T_c$ becomes $121T_c^2$. Thus, a numerically better approximation is possible, however, the choice of z_m does not give a big qualitative difference.

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